

Free Vibrations of Nanoscale Beam Under Two-Temperature Green and Naghdi Model

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The influence of the coupling between strain rate and temperature becomes domineering in the nanoscale beam. In this work, the free vibration of nanoscale beam resonators is analysed using Green and Naghdis theorem under the two-temperature model (2TGNIII). The influence of two-temperature parameters in a nanoscale beam is studied for beams under simply supported conditions. Exact expressions for frequency shift and the thermoelastic damping have been derived in the resonator, and calculation outcomes have been presented graphically with respect to frequency shift, natural frequency, and thermoelastic damping. The scale of length and thickness for a nanobeam equal to 15×10^{-9} m and equal to 1.3×10^{-12} s for time.

1. INTRODUCTION

Many attempts have been made to study the elastic properties of nanostructured materials by atomistic simulations. Diao et al.¹ investigated the influence of free surface on the body and elastic properties of gold nanowires using atomic emulations. Modelling and emulation of thermoelastic damping is a subject of repeated attention in the nanomechanics community and nanoengineering community, one most encouraged by nanoelectromechanical advancement system (NEMS) technologies. The systems of Nanoelectromechanical, or NEMS, reach quite high essential frequencies of procedure, especially when one considers their miniature size and small force constants. Such mechanical devices of high frequencies have many significant applications, among which are scanning probe microscopes, mechanical signal processing, and ultrasensitive mass detection.

Lord and Shulman² extended the couple thermoelastic theory. Green and Lindsay³ included the thermal relaxation times in constitutive equations. The counterparts of our problems in the context of theories of thermoelastic theory were considered by using numerical and analytical approaches.⁴⁻⁹ Green and Naghdi^{10,11} established GNII and GNIII generalized thermoelastic models, which based the replacing of usual entropy inequality alongside the entropy equality. In recent years, various problems have been taken into account using the Green and Naghdi models.¹²⁻²⁶

Thermoelasticity using two-temperature modelling is one of the unconventional thermoelastic models of elastic solids. Thermal dependence is the main variance from this theorem compared to classical theory. Chen et al.²⁷⁻²⁹ established a theorem of thermal conduction in deformable bodies, which based on two featured temperatures: thermodynamic and conductive. The variance between these two temperatures is proportionate to the thermal supply for time independent cases. For time-dependent problems and problems of waves propagation in particular, the two temperatures are mostly different in cases of the presence and absence of the thermal input. Youssef³⁰ presented the generalized thermoelastic theory under two-temperature by using Fourier law to the field equations. El-Karamany and Ezzat³¹ introduced the two-temperature Green-Naghdi thermoelasticity models. Abbas et al.³²⁻³⁴ presented various problems based on the two-temperature thermoelastic model using numerical and analytical methods.

Due to their numerous significant technological applica-

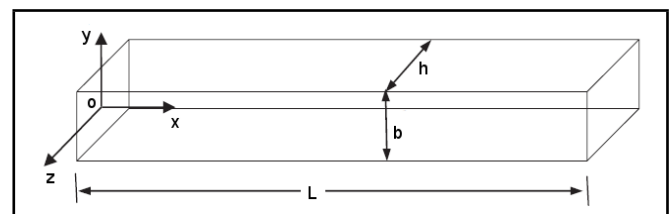


Figure 1. Schematic illustration of the beam setup.

tions, nanomechanical resonators have attracted considerable attention. The processes of vibration and heat transfer have been studied by many researchers. Honsten et al.³⁵ predicted that the inner friction in 50-nm scale silicon-based MEMS structures is strong when subjected to thermoelastic damping. Nayfeh and Younis³⁶⁻³⁹ presented analytical expressions for the quality factor of microplates of general shapes subjected to thermoelastic damping. Rezazadeh et al.⁴⁰ used the modified couple stress model to study the thermoelastic damping in a microbeam resonator. Sun et al.⁴¹ investigated thermoelastic damping in microbeam resonators. Elsibai and Youssef⁴² applied the state-space method to study the vibration of the gold nanobeam due to ramp-type heating under Green and Naghdis type II model. Sharma⁴³ investigated the frequency shift and thermoelastic damping in both microscale and nanoscale anisotropic resonators. Grover and Sharma^{44,45} studied the effect of relaxation time for thermoelastic vibration on MEMS and NEMS with voids.

This paper investigates the free vibrations of a nanobeam resonator in the context of the Green and Naghdi theory with two-temperature (2TGNIII). The impact of two-temperature parameters in the natural frequency, frequency shift, and the thermoelastic damping have been studied and are graphically represented.

2. FORMULATIONS OF THE PROBLEM

We consider the theoretical analysis of small flexural deflection of an isotropic, homogenous heat conductor, thermoelastic resonator by using the Cartesian coordinate system xyz for the temperature increment $T(x, y, z, t)$ and the vector of displacement $u(x, y, z, t) = (u, v, w)$, which have the dimension thickness h ($-\frac{h}{2} \leq z \leq \frac{h}{2}$), the width b ($-\frac{b}{2} \leq y \leq \frac{b}{2}$), and the length L ($0 \leq x \leq L$), as in Fig. 1. In short, any plane

cross-section that is initially perpendicular to axis of beam remains plane and perpendicular to the neutral surface during bending. So, the components of displacement can be expressed by:

$$u = -z \frac{\partial w}{\partial x}, \quad v = 0, \quad w(x, y, z, t) = w(x, t). \quad (1)$$

Then, the motion equation in the absence of pressures on the lower and the upper surface of the beam is given by⁴⁶:

$$\frac{\partial^2 M}{\partial x^2} + \rho A \frac{\partial^2 w}{\partial t^2} = 0; \quad (2)$$

where ρ is the density of the medium, t is the time, $A = bh$ is the area of cross-section, and M is the flexural moment of cross-section of beam.

$$\begin{aligned} M(x, t) &= ; \\ &= - \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} (-(\lambda + 2\mu) \frac{\partial^2 w}{\partial x^2} - \gamma T) z dz dy = ; \\ &= (\lambda + 2\mu) I \frac{\partial^2 w}{\partial x^2} + \gamma M_T; \end{aligned} \quad (3)$$

where $I = \frac{bh^3}{12}$ is the moment of inertia of the cross-section, λ, μ are the Lamé's constants, $T = T^* - T_0$ is the thermodynamic temperature deviation from the reference temperature T_0 , $\gamma = (2\lambda + 3\mu)\alpha_t$ and α_t is the coefficient of linear thermal expansion, and M_T is the beam thermal moment which takes the form:

$$M_T = \int_{-\frac{h}{2}}^{\frac{h}{2}} b T z dz. \quad (4)$$

The equation of heat conduction can be written by:

$$\begin{aligned} K^* \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} \right) + K \left(\frac{\partial^3 \varphi}{\partial t \partial x^2} + \frac{\partial^3 \varphi}{\partial t \partial z^2} \right) &= ; \\ &= \frac{\partial^2}{\partial t^2} (\rho c_e T - \gamma T_0 z \frac{\partial^2 w}{\partial x^2}); \end{aligned} \quad (5)$$

where K is the thermal conductivity, K^* is the material constant characteristic of the theory, $\varphi = \varphi^* - T_0$ is the conductive temperature deviation from the reference temperature T_0 , and c_e is the specific heat at constant strain. The relation between the conductive and thermodynamic temperatures is given by³¹:

$$T = \varphi - \alpha \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} \right); \quad (6)$$

where $\alpha > 0$ is the two-temperature parameter. To solve this problem, the harmonic solution can be expressed by:

$$[w(x, t), T(x, z, t)] = [w(x), T(x, z)] e^{i\omega t}. \quad (7)$$

The preceding governing equations can be put in non-dimensional forms using the following dimensionless parameters:

$$\begin{aligned} (x', y', z', w') &= \frac{(x, y, z, w)}{c\chi}; \quad M'_T = \frac{M_T}{T_0 c^3 \chi^3}; \\ T' &= \frac{T}{T_0}; \quad \varphi' = \frac{\varphi}{T_0}; \quad \omega' = \omega \chi; \quad t' = \frac{t}{\chi}; \quad \alpha' = \frac{\alpha}{c^2 \chi^2}; \end{aligned}$$

where,

$$c^2 = \frac{\lambda + 2\mu}{\rho}; \quad \chi = \frac{K}{\rho c_e c^2}. \quad (8)$$

Thus, (when the primes have been dropped for convenience) the above equations in non-dimensional forms can be simplified by:

$$I \frac{\partial^4 w}{\partial x^4} + \frac{\gamma T_0}{\lambda + 2\mu} \frac{\partial^2 M_T}{\partial x^2} - \omega^2 A w = 0; \quad (9)$$

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} = \frac{-\omega^2}{\varepsilon - \alpha \omega^2 + i\omega} \left(\varphi - \frac{\gamma}{\rho c_e} z \frac{\partial^2 w}{\partial x^2} \right). \quad (10)$$

2.1. Application

We suppose that the material is initially at rest. At reference temperature, the undisturbed state has been maintained. Therefore, one obtains the equation:

$$\begin{aligned} w(x, 0) &= \frac{\partial w(x, 0)}{\partial t} = 0; \\ \varphi(x, z, 0) &= \frac{\partial \varphi(x, z, 0)}{\partial t} = 0. \end{aligned} \quad (11)$$

These conditions are completed by considering that the two ends of the nanoscale are simply supported. Thus, the boundary conditions can be written by:

$$\begin{aligned} w(0, t) &= w(L, t) = 0; \\ \frac{\partial^2 w(0, t)}{\partial x^2} &= \frac{\partial^2 w(L, t)}{\partial x^2} = 0. \end{aligned} \quad (12)$$

We consider the case of there is no flow of heat across the lower and upper surfaces of the nanobeam, which gives:

$$\frac{\partial \varphi(x, -\frac{h}{2}, 0)}{\partial z} = \frac{\partial \varphi(x, \frac{h}{2}, 0)}{\partial z} = 0. \quad (13)$$

2.2. Solution Along Thickness Direction

We follow the same procedures.⁴⁶ Noting that temperature gradients in the plane of the cross section along z direction are much larger than those along the x direction and that no gradients exist in the y direction, we can replace Eq. (10) by:

$$\frac{\partial^2 \varphi}{\partial z^2} = \frac{-\omega^2}{\varepsilon - \alpha \omega^2 + i\omega} \left(\varphi - \frac{\gamma}{\rho c_e} z \frac{\partial^2 w}{\partial x^2} \right); \quad (14)$$

where $\varepsilon = \frac{K^*}{\rho c_e c^2}$ because there is no flow of heat across the lower and upper surfaces of the beam. Then the general solution of Eq. (14) takes the form:

$$\varphi(x, z) = \frac{\gamma}{\rho c_e} \left(z - \frac{\sin(pz)}{p \cos(\frac{ph}{2})} \right) \frac{\partial^2 w}{\partial x^2}; \quad (15)$$

where $p = \sqrt{\frac{\omega^2}{\varepsilon - \alpha \omega^2 + i\omega}}$. Substituting Eqs. (15) and (6) with Eqs. (4) and (8) in Eq. (10), we get:

$$D_\omega \frac{\partial^4 w}{\partial x^4} - \omega^2 w = 0; \quad (16)$$

where $D_\omega = \frac{I}{A} (1 + \varepsilon_T [(1 + (1 + \alpha p^2) f(\omega))])$, $\varepsilon_T = \frac{\gamma^2 T_0}{\rho c_e (\lambda + 2\mu)}$ and $f(\omega) = \frac{24}{\rho^3 h^3} (\frac{ph}{2} - \tan \frac{ph}{2})$. From Eq. (16) we can drive the frequency of vibration in the presence of the two-temperature parameter α and the thermoelastic coupling ε_T :

$$\omega_m = \frac{m^2 \pi^2}{L^2} \sqrt{D_\omega} = \omega_0 \sqrt{1 + \varepsilon_T [1 + (1 + \alpha p^2) f(\omega)]}; \quad (17)$$

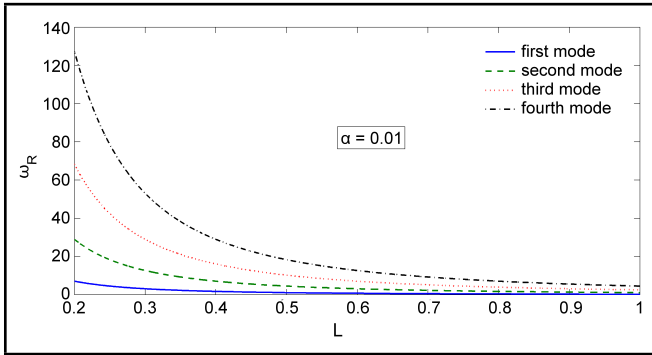


Figure 2. Non-dimensional frequency ω_R versus the length L .

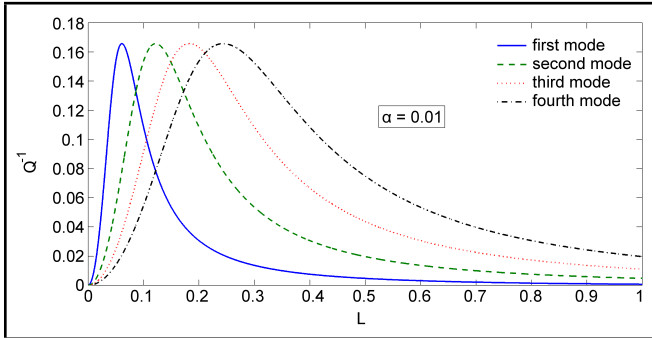


Figure 3. Thermoelastic damping Q^{-1} versus the length L .

where $\omega_0 = \frac{hm^2\pi^2}{L^2\sqrt{12}}$. For most of the material $\varepsilon_T \ll 1$, we can replace ω with ω_0 and $f(\omega)$ with $f(\omega_0)$ to get:

$$\omega^m = \omega_0 \sqrt{1 + \varepsilon_T [1 + (1 + \alpha p^2) f(\omega_0)]}. \quad (18)$$

The thermoelastic damping can be expressed by:

$$Q^{-1} = 2 \left| \frac{\omega_I^m}{\omega_R^m} \right|; \quad (19)$$

where ω_I^m and ω_R^m are the imaginary and real parts of frequency ω^m , and m is the mode number, which corresponds to the transcendental equation roots in Eq. (18). The frequency shift due to thermal variations is defined as:

$$\omega_S = \left| \frac{\omega_R^m - \omega_0}{\omega_0} \right|. \quad (20)$$

3. NUMERICAL RESULTS AND DISCUSSION

Now, we will propose a numerical example for which computational outcomes are given. For this, gold (Au) has been taken as the thermoelastic medium for which we take the physical parameters by the following values:⁴²

$$\begin{aligned} \lambda &= 1.98 \times 10^{11} \text{ Nm}^{-2}; \\ \mu &= 0.27 \times 10^{11} \text{ Nm}^{-2}; \\ T_0 &= 293 \text{ K}; \\ \rho &= 1930 \text{ kgm}^{-3}; \\ c_e &= 130 \text{ Jkg}^{-1} \text{ K}^{-1}; \\ \alpha_t &= 14.2 \times 10^{-6} \text{ K}^{-1}. \end{aligned}$$

Numerical computations are carried out for two cases, when $h = 0.1$ and when $0 < L < 1$. The first case is studying how the dimensionless frequency ω_R , the thermoelastic

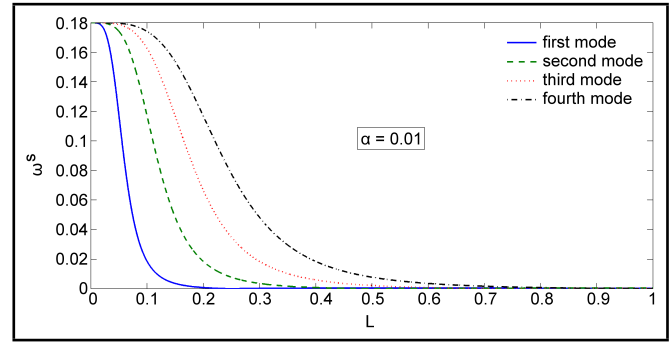


Figure 4. Frequency shift ω_S versus the length L .

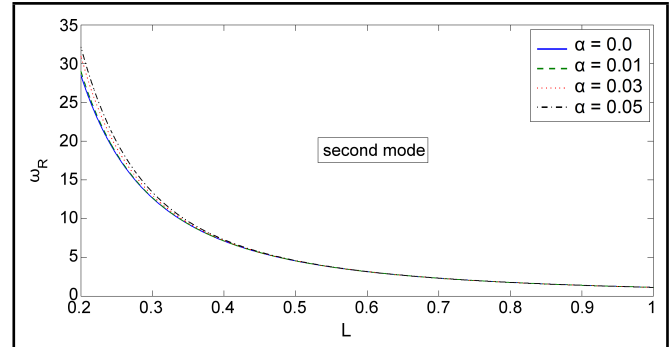


Figure 5. Non-dimensional frequency ω_R versus the length L for different values of α .

damping Q^{-1} , and the frequency shift ω_S vary with various modes when the parameter of two-temperature ($\alpha = 0.01$) remains constant. The second is studying how the dimensionless frequency ω_R , the thermoelastic damping Q^{-1} and frequency shift ω_S vary with various values of parameter of two-temperature for the second mode. The numerical outcomes are obtained and graphically presented in Figs. 2–7. Figure 2 shows the variations of dimensionless frequency ω_R versus the length L for the first four modes when the parameter of two-temperature ($\alpha = 0.01$) remains constant. It is observed that the dimensionless frequency ω_R reduces as the length L increases. Figure 3 exhibits the thermoelastic damping Q^{-1} versus the length L for the first four modes when the parameter of two-temperature ($\alpha = 0.01$) remains constant. It is noticed that the thermoelastic damping Q^{-1} rises initially to attain the highest peak values before it reduces in order to become ultimately asymptotic with rising L . Figure 4 shows the behavior of the frequency shift ω_S versus length L for the first four modes when the two-temperature parameter ($\alpha = 0.01$) remains constant. It can be deduced that the frequency shift ω_S starts from the maximum value from the first end of the beam and then decreases rapidly when increasing the length to zero for large values of length. Figures 5, 6, and 7 show the variation of the natural frequency ω_R , thermoelastic damping Q^{-1} , and the frequency shift ω_S respectively, versus L for different values of the two-temperature parameter. Thus, important phenomena are observed that the two-temperature parameter has a great effect on the distribution of field quantities.

4. CONCLUSIONS

The free vibration analysis of generalized thermoelastic nanoscale resonators in the context of Green and Naghdis two-temperature model (2TGNIII) has been carried out. The exact solutions of frequency shifts, thermoelastic damping, deflection, and the temperature increment in the nanoscale resonator have been introduced. The exact solutions obtained here pave

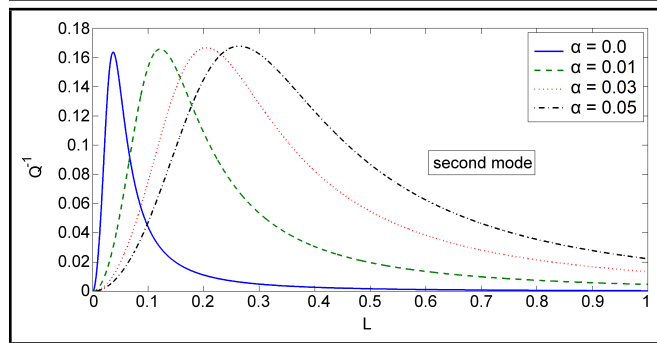


Figure 6. Thermoelastic damping Q^{-1} versus the length L for different values of α .

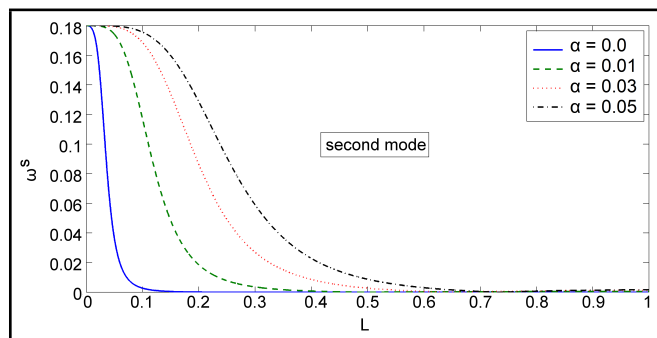


Figure 7. Frequency shift ω_s versus the length L for different values of α .

the way for further investigations in engineering, mathematics, and science.

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